

Power Suppressed Corrections to Hadronic Event Shapes

Aneesh V. Manohar

Department of Physics, University of California at San Diego, La Jolla, CA 92093

Mark B. Wise

California Institute of Technology, Pasadena, CA 91125

Abstract

For high energy processes ($M \gg \Lambda_{QCD}$) there are infrared safe hadronic shape variables that have a calculable perturbative expansion in $\alpha_s(M^2)$. However, nonperturbative power suppressed corrections to these variables are not well understood. We use the behavior of large orders of the perturbation expansion to gain insight into the nonperturbative corrections. Our results suggest that certain shape variables have nonperturbative corrections suppressed by fractional powers of Λ_{QCD}^2/M^2 .

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Many large momentum transfer processes involving the strong interactions have a power series expansion in the strong coupling constant $\alpha_s(M^2)$. In addition to this perturbative expansion in α_s , there are nonperturbative corrections suppressed by powers of $\Lambda_{\text{QCD}}^2/M^2$, where M is the typical momentum transfer, $M \gg \Lambda_{\text{QCD}}$. In some cases the nonperturbative corrections are well understood. For example, in $R(M^2) = \sigma(e^+e^- \rightarrow \text{hadrons})/\sigma(e^+e^- \rightarrow \mu^+\mu^-)$, with $M^2 = (p_{e^-} + p_{e^+})^2$, the operator product expansion can be used to show that nonperturbative power corrections are proportional to the vacuum expectation values of gauge invariant local operators.¹ The leading nonperturbative correction is of order $\Lambda_{\text{QCD}}^4/M^4$, and arises from the expectation value of the dimension four operator $\text{Tr } G_{\mu\nu} G^{\mu\nu}$, where $G_{\mu\nu}$ is the gluon field strength tensor. The operator product expansion shows that there are no corrections of order $\Lambda_{\text{QCD}}^2/M^2$, since there is no gauge invariant dimension two operator that can be constructed out of quark and gluon fields. However, there are physical quantities that can be predicted using perturbative QCD for which the nonperturbative corrections are not well understood. For example, infrared safe event shape variables in $e^+e^- \rightarrow \text{hadrons}$ have a perturbative expansion in $\alpha_s(M^2)$. Since these variables weight the final hadronic states in a way that depends on their shape, they are not given by the imaginary part of the time ordered product of electromagnetic currents and an operator product expansion does not seem feasible. In this letter we examine some features of the nonperturbative corrections to hadronic shape variables in a toy model.

The strong interaction coupling constant $\alpha_s(M^2)$ falls logarithmically with the momentum transfer M^2 . Nevertheless, perturbative QCD corrections can also produce power suppressed corrections [2] to physical quantities, because the QCD perturbative expansion is an asymptotic series. Information on the power suppressed corrections to physical quantities can be obtained from an examination of the behavior of the perturbative expansion in α_s at large orders. Consider a physical quantity $P(\alpha_s)$ that has a power series expansion

$$P(\alpha_s) = \sum_{n=0}^{\infty} p_n \alpha_s^{n+1} \quad (1)$$

in the strong coupling $\alpha_s(M^2)$. Its Borel transform is defined by

$$B[P](t) = \sum_{n=0}^{\infty} \frac{p_n t^n}{n!}. \quad (2)$$

¹ There may also be non-perturbative instanton contributions to the coefficients of operators [1]

The Borel transform $B[P](t)$ may converge, even if the original series $P(\alpha_s)$ is only an asymptotic expansion. This provides a definition of $P(\alpha_s)$ through the integral transform

$$P(\alpha_s) = \int_0^\infty dt e^{-(t/\alpha_s)} B[P](t). \quad (3)$$

However, if the coefficients p_n grow too fast with n , the Borel transform $B[P]$ will have singularities that prevent using (3) to define $P(\alpha_s)$. In an asymptotically free theory such as QCD, singularities in $B[P](t)$ for positive t are associated with the infrared properties of Feynman diagrams, and are usually referred to as infrared renormalons. One can still obtain $P(\alpha_s)$ from $B[P](t)$ using the inverse Borel transform eq. (3), provided one deforms the contour of integration in eq. (3) away from the real t axis, to avoid the renormalon singularity. The resultant expression for $P(\alpha_s)$ is no longer unique (or even real), and depends on whether the deformed path passes above or below the renormalon pole. Suppose, for example, that $B[P](t)$ has a simple pole at

$$t = \frac{u}{b_0}, \quad (4)$$

where b_0 is the first term in the β -function (negative in QCD),

$$\mu^2 \frac{d\alpha_s}{d\mu^2} = b_0 \alpha_s^2 + \mathcal{O}(\alpha_s^3), \quad (5)$$

which governs the large M^2 behavior of the coupling constant in an asymptotically free theory,

$$\alpha_s(M^2) \sim \frac{1}{(-b_0) \ln(M^2/\Lambda_{\text{QCD}}^2)}, \quad (6)$$

and $u < 0$ is a constant (which is usually, but not always, an integer). The magnitude of the ambiguity in eq. (3) associated with this singularity can be estimated by the difference between the value of P obtained by deforming the t integration contour above and below the pole [3]. The difference is

$$\begin{aligned} |\Delta P| &\sim e^{-u/b_0 \alpha_s} \\ &\sim (\Lambda_{\text{QCD}}^2/M^2)^{(-u)}. \end{aligned} \quad (7)$$

Thus infrared renormalons produce power suppressed ambiguities in P . The infrared renormalon closest to the origin $u = t = 0$ gives the dominant ambiguity. For $R(M^2)$, the total cross-section for $e^+e^- \rightarrow \text{hadrons}$ defined previously, it is believed that the

perturbative expansion is not Borel summable, and has a renormalon at $u = -2$, which gives rise to an ambiguity in R that is of order $\Lambda_{\text{QCD}}^4/M^4$ [4][5]. This ambiguity is of the same form as the leading non-perturbative correction in the operator product expansion for $R(M^2)$, due to the vacuum expectation value of $\text{Tr } G_{\mu\nu}G^{\mu\nu}$. It is thought that there is an ambiguity in the definition of $\langle 0 | \text{Tr } G_{\mu\nu}G^{\mu\nu} | 0 \rangle$, and that the renormalon ambiguity in the sum of the perturbation series cancels the ambiguity in the non-perturbative matrix element, to give a well-defined result for the total cross-section $R(M^2)$ [6].

Renormalon ambiguities can only be absorbed into non-perturbative matrix elements if the contributions of both quantities to a physical observable such as $R(M^2)$ have the same M dependence. Renormalon ambiguities can be absorbed in gauge invariant matrix elements provided the operator dimension is equal to $-2u$. Explicit computations on renormalons are at a primitive level. Typically, one sums bubble graphs like those given in fig. 1 to determine the renormalon singularities [7]. Other graphs which have been neglected are presumably just as important, so the result of summing the bubble chain is not a proof of the existence of renormalon effects. Renormalon effects have been computed by Beneke [8] in a limiting case of QCD. One considers QCD with N_f flavors, and takes the limit $N_f \rightarrow \infty$ with $a = N_f \alpha_s$ held fixed. Feynman diagrams are computed to leading order in α_s , but to all orders in a . Terms in the bubble sum of fig. 1 with any number of bubbles are kept in this limit, since each additional fermion loop contributes a factor $\alpha_s N_f$, which is not treated as small. The $N_f \rightarrow \infty$ limit has its limitations, since QCD is not an asymptotically free theory in this limit. The procedure used by Beneke is to write the Borel transform as a function of $u = b_0 t$, (where b_0 is now positive), but still study renormalons for negative u . The singularities at negative u are then taken to be the infrared renormalons for asymptotically free QCD. This procedure was used by Beneke to study renormalons in $R(M^2)$. It has been suggested by Brown and Yaffe [9] that there might be a renormalon at $u = -1$, which would lead to an order $\Lambda_{\text{QCD}}^2/M^2$ ambiguity in the perturbative expansion for $R(M^2)$ that could not be compensated by the matrix element of a local operator. Beneke found by explicit computation that the renormalon singularity at $u = -1$ vanished.

In this letter we examine the behavior of shape variables at large orders in perturbation theory using a toy model. The model consists of a $U_1(1) \times U_2(1)$ gauge group (with gauge couplings e_1 and e_2), and a large number N_f of massless fermions f with charge (1,0).

We suppose there is a neutral scalar ϕ of mass M that decays to the massless degrees of freedom through the dimension five interaction Lagrangian density.

$$\mathcal{L}_{\text{int}} = \lambda e_1 e_2 \phi F_{\mu\nu}^{(1)} F^{(2)\mu\nu} \quad (8)$$

where λ has mass dimension -1 . We will compute the energy-energy correlation [10] for ϕ decay to leading order in α_1 and α_2 , and to all orders in $a = N_f \alpha_1$. Physical situations analogous to this exist in QCD. For example a color singlet (Higgs) scalar ϕ could decay to hadronic final states via the interaction $\mathcal{L}_{\text{int}} = \lambda g^2 \phi \text{Tr } G_{\mu\nu} G^{\mu\nu}$. The non-abelian structure of QCD plays no role in the bubble chain sum of fig. 1, so we have simplified the computation by choosing an Abelian gauge theory, with two different $U(1)$'s.

The total ϕ decay rate can be computed using the operator product expansion, and the computation is very similar to the computation of $R(M^2)$. The leading operator is 1, and the first correction comes from dimension four operators. One can also compute the Borel singularities using the methods of ref. [8]. It is straightforward to show that the total decay rate has no singularities in the Borel plane.

A more interesting computation is that of event shapes in ϕ decay. There is no operator product expansion for the event shapes, so it is interesting to see whether there exist renormalon singularities, and whether they are at integer values of u . A convenient characterization of event shapes in ϕ decay is the energy-energy correlation [10]

$$\begin{aligned} \mathcal{P}(\cos \chi) = & \sum_{ij} \int \frac{d^3 \Gamma}{dE_i dE_j d\cos \theta_{ij}} \left(\frac{E_i}{M} \right) \left(\frac{E_j}{M} \right) \delta(\cos \theta_{ij} - \cos \chi) dE_i dE_j d\cos \theta_{ij} \\ & + \delta(1 - \cos \chi) \sum_i \int \frac{d\Gamma}{dE_i} \left(\frac{E_i}{M} \right)^2 dE_i. \end{aligned} \quad (9)$$

In eq. (9), the double sum is over all pairs ij of particles in the final state with $i \neq j$, and ij and ji are both included in the sum. The angle between the three-momentum vectors of particles i and j in the ϕ rest frame is denoted by θ_{ij} , and E_i represents the energy of particle i . The Fox–Wolfram moments [11] H_L are obtained as integrals of the energy-energy correlations with Legendre polynomials,

$$\int_{-1}^1 d\cos \chi \mathcal{P}(\cos \chi) P_L(\cos \chi) = \Gamma H_L, \quad (10)$$

where Γ is the total ϕ decay rate. Integrals of $\mathcal{P}(\cos \chi)$ against any “smooth” weighting function $w(\cos \chi)$ are infrared safe event shape variables.

The Feynman diagrams of fig. 2 contribute to the ϕ decay distribution to lowest order in λ, α_1 and α_2 , and to all orders in $a = N_f \alpha_1$. We will restrict our analysis to shape variables that can be obtained by integrating \mathcal{P} against a weighting function that vanishes at $\cos \chi = \pm 1$. This simplifies the calculation because the Feynman diagram in fig. (2a) and the single sum in eq. (9) don’t contribute. The remaining Feynman graphs are easy to compute, and one obtains

$$\mathcal{P}(\cos \chi) = \int_0^{M^2} dq^2 \frac{d\mathcal{P}(\cos \chi)}{dq^2}, \quad (11)$$

where

$$\frac{d\mathcal{P}(\cos \chi)}{dq^2} = \frac{2M\lambda^2\alpha_2}{N_f} \frac{a^2}{|1 + \Pi(q^2)|^2} F(q^2, \cos \chi). \quad (12)$$

Here q^2 is the invariant mass of the fermion pair,

$$\Pi(q^2) = -a b_0 \ln(-q^2/\Lambda^2), \quad b_0 = 1/3\pi, \quad (13)$$

is the fermion bubble contribution to the gauge propagator, and

$$F(q^2, \cos \chi) = \frac{\hat{q}^2 (1 - \hat{q}^2)^4 \left[(1 + \cos \chi)^2 + \hat{q}^4 (1 - \cos \chi)^2 \right]}{[(1 + \cos \chi) + \hat{q}^2 (1 - \cos \chi)]^5} + \frac{\theta \left((1 + \hat{q}^2)^2 (1 - \cos \chi) - 8\hat{q}^2 \right) \hat{q}^2 \left[(1 - \cos \chi) (1 + \hat{q}^4) - 4\hat{q}^2 \right]}{(1 - \cos \chi)^{7/2} \left[(1 + \hat{q}^2)^2 (1 - \cos \chi) - 8\hat{q}^2 \right]^{1/2}}, \quad (14)$$

with $\hat{q}^2 = q^2/M^2$. In eq. (14) terms proportional to $\delta(1 + \cos \chi)$ and $\delta(1 - \cos \chi)$ are neglected. The quantity Λ in eq. (13) is proportional to the subtraction point μ used to define the finite part of the vacuum polarization, Π , and is in general subtraction scheme dependent.

We are interested in the Borel transform of $\mathcal{P}(\cos \chi)$ with respect to the variable $a = N_f \alpha_1$. The dependence of $\mathcal{P}(\cos \chi)$ on a is in the function

$$g(a) = \frac{a^2}{|1 + \Pi(q^2)|^2} = \frac{a^2}{(1 - ab_0 \ln(q^2/\Lambda^2))^2 + b_0^2 a^2 \pi^2},$$

$$= \sum_{m=0}^{\infty} \frac{(-1)^m \pi^{2m}}{(2m+1)!} \sum_{n=2m}^{\infty} (n+1) n \dots (n-2m+1) b_0^n a^{n+2} (\ln(q^2/\Lambda^2))^{n-2m}.$$

((15)(15)equation15equation1515)

The Borel transform of $g(a)$ is

$$\begin{aligned}
B[g](t) &= \sum_{m=0}^{\infty} \frac{(-1)^m \pi^{2m}}{(2m+1)!} \sum_{n=0}^{\infty} \frac{1}{n!} b_0^{n+2m} t^{n+2m+1} (\ln(q^2/\Lambda^2))^n, \\
&= (q^2/\Lambda^2)^{b_0 t} \sin(\pi b_0 t), \\
&= (q^2/\Lambda^2)^u \sin(\pi u),
\end{aligned}
\tag{(15)(15)equation16equation1616}$$

using $u = b_0 t$.

Consider an infrared safe shape variable S_η that corresponds to a weighting function $w(\cos \chi)$ that behaves like $w(\cos \chi) \sim (1 + \cos \chi)^\eta$ near $\cos \chi = -1$. The singularities in the Borel transform of this shape variable that arise from the integration region near $\cos \chi = -1$ are determined by the value of $F(q^2, \cos \chi)$ for small q^2 and $\cos \chi$ near -1 . In this region

$$F(q^2, \cos \chi) \sim \frac{\hat{q}^2 \left[(1 + \cos \chi)^2 + 4\hat{q}^4 \right]}{\left[(1 + \cos \chi) + 2\hat{q}^2 \right]^5}.
\tag{(15)(15)equation17equation1717}$$

Using eqs. (11)–(17) we find that the Borel transform of S_η has infrared renormalons. For $\eta < 1$ the singularity closest to the origin is a simple pole at

$$u = -\eta \quad \Rightarrow \quad t = -\eta/b_0.
\tag{(15)(15)equation18equation1818}$$

For $\eta = 1$ there is potentially a simple pole at $u = -1$. However the $\sin(\pi u)$ factor cancels the pole at $u = -1$, and there is no singularity in $B[S_1]$ at $u = -1$. The singularity closest to the origin is a simple pole at $u = -2$.

An integral of the form

$$\int_0^{M^2} dq^2 h(q^2) B[g] = \sin(\pi u) \int_0^{M^2} dq^2 h(q^2) (q^2/\Lambda^2)^u
\tag{(15)(15)equation19equation1919}$$

determines the singularities in $B[S_\eta]$. If $h(q^2) \sim (q^2)^k$ for small q^2 , there is no singularity provided k is an integer, and one obtains a simple pole singularity at $u = -k-1$ if k is not an integer. There is a simple pole singularity at $u = -k-1$ for integer k if $h(q^2) \sim (q^2)^k \ln(q^2)$ for small q . The weighting function $(1 + \cos \chi)^\eta$ produces an effective $(q^2)^{\eta-1}$ behavior² for $h(q^2)$ at small q^2 , and hence has a simple pole singularity at $u = -\eta$. For $\eta = 1$ (or

²⁰footnote22 Although $h(q^2)$ is singular as $q^2 \rightarrow 0$, it is integrable so that S_η is infrared safe.

more precisely, for $w(\cos \chi) = \cos^2 \chi - 1$ which vanishes at $\cos \chi = \pm 1$), $h(q^2)$ in eq. (19) has the form $c_0 + c_1 q^2 \ln q^2$ for small q^2 , where c_0 and c_1 are constants. The absence of a $\ln q^2$ term (which is infrared safe) implies that there is no renormalon at $u = -1$. The $q^2 \ln q^2$ term produces the pole at $u = -2$. In more realistic examples, one expects that there will be a $\ln q^2$ term, and hence a renormalon singularity at $u = -1$ as well.

The behavior of QCD hadronic event shape variables can be inferred from the toy example using the method of Beneke [8]. Our results imply that Fox–Wolfram moments will have renormalons at integer values of u . Event shape variables defined by weighting functions that are polynomials in $\cos \chi$ (e.g. Fox–Wolfram moments) have Borel transforms that are more singular than the Borel transform of the total rate. More interesting are the shape variables S_η ($0 < \eta < 1$) which have a renormalon at $u = -\eta$. These variables have ambiguities of order $(\Lambda_{QCD}^2/M^2)^\eta$ from the high-order sum of the perturbation expansion. Non-perturbative power suppressed corrections are expected to be of the same order and cancel this ambiguity, leaving a residual power suppressed correction. The unusual power dependence of the shape variables S_η arises from weighting functions $w(\cos \chi)$ that are continuous but not differentiable at $\cos \chi = -1$. Our results indicate that hadronic shape variables which are infrared safe but do not treat the region where infrared divergences cancel between Feynman diagrams in a very smooth fashion can have large power suppressed corrections. We hope to present results for more realistic situations in a future publication.

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References

- on20reference11] M.A. Shifman, A.I. Vainshtein, V.I. Zakharov, Nucl. Phys. B147 (1979) 385
- on20reference22] V.I. Zakharov, Nucl. Phys. B385 (1992) 452
- on20reference33] J. Zinn-Justin, Phys. Rep. 49 (1979) 205
- on20reference44] G. Parisi, Phys. Lett. B76 (1978) 65, Nucl. Phys. B150 (1979) 163
- on20reference55] A.H. Mueller, Nucl. Phys. B250 (1985) 327
- on20reference66] F. David, Nucl. Phys. B209 (1982) 433, Nucl. Phys. B234 (1984) 237, Nucl. Phys. B263 (1986) 637
- on20reference77] G. 't Hooft, in *The Whys of Subnuclear Physics*, edited by A. Zichichi (Plenum, New York, 1978)
- on20reference88] M. Beneke, Nucl. Phys. B405 (1993) 424
- on20reference99] L.S. Brown and L.G. Yaffe, Phys. Rev. D45 (1992) R398 ;
L.S. Brown, L.G. Yaffe and C. Zhai, Phys. Rev. D46 (1992) 4712
- 20reference1010] C.L. Basham, L.S. Brown, S.D. Ellis, and S.T. Love, Phys. Rev. Lett. 41 (1978) 1585
- 20reference1111] G.C. Fox and S. Wolfram, Phys. Rev. Lett. 41 (1978) 1581

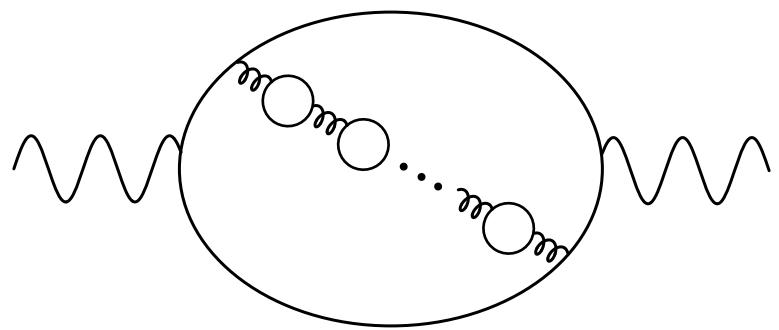
Figure Captions

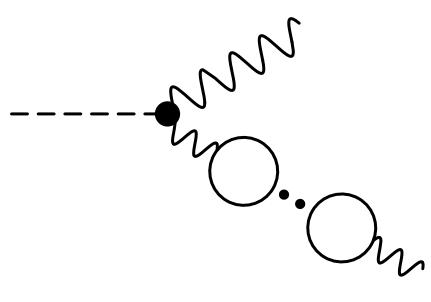
5) equation 20 figure 11. Graphs that give an infrared renormalon in QCD.

5) equation 20 figure 22. Feynman diagrams contributing to the event shape distribution to lowest order in λ , α_1 and α_2 , and to all orders in $N_f \alpha_1$.

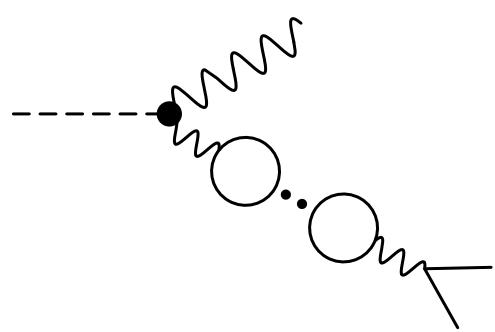
Figure 1

Figure 2





(a)



(b)